Limited Liability, Corporate Value, and the Demand for Liability Insurance

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ABSTRACT

This article models a corporation characterized by a positive probability of insolvency, in a financial market setting. The analysis shows that the positive insolvency probability separates the private from the social costs of the firm's operations. It shows that, *ceteris paribus*, purchasing liability insurance will not create value but will shift value between claimholders. The insurance appears to change value because the corporate value does not fully reflect the value of all the stakeholders' claims. The analysis also shows when management has an incentive to purchase insurance and that the insurance eliminates the difference between private and social costs.

Introduction

Limited liability plays an important role in financial markets by enabling corporations to raise a sufficient amount of money to finance risky investments. It may, however, also create some difficulties for the operation of a competitive financial market system. If there is a positive probability that the firm will become insolvent, even in the absence of a risky bond issue, then limited liability protects shareholders but it also separates the private from the social costs of the firm's operations. Limited liability allows stockholders to walk away from corporate liabilities when earnings are insufficient to cover those liabilities. Hence, the stockholders may be viewed as holding a put option which allows them to put the firm to the liability claimants[1] and other general creditors in the event of insolvency.[2] The general creditors become the

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This research was partially funded by the Gus Wortham Chair of Insurance and Risk Management. We thank Robert Witt, Travis Pritchett and two anonymous referees for their comments on earlier drafts of this paper.

[1] The terms liability claimants, tort claimants, and involuntary creditors will be used synonymously here. This body of claimants is a subset of the group of general creditors.

[2] Viewing stockholders as holding a put option is not new, e.g., see Black and Scholes (1983). The expanded scope of the corporation's contract set provided here, however, does show that it is

owners and share the liquidation value. The loss to liability claimants generates a social cost for the economy. This problem can be dealt with through public measures or through the creation of market institutions such as insurance companies. The purpose of this article is first, to analyze the impact of liability insurance on the value of the stakeholders positions in the firm and second, to demonstrate the conditions under which this insurance is demanded.

The claim that limited liability can separate the private from the social cost of corporate operations has been made by Easterbrook and Fischel (1985). According to Easterbrook and Fischel,

> When corporations must pay for the right to engage in risky activities, they will tend to undertake projects only where social benefits equal social costs at the margin. Where high transactions costs prohibit those affected by risky activities from charging an appropriate risk premium, however, the probability that firms with limited liability will undertake projects with an inefficiently high level of risk increases. Firms capture the benefits from such activities while bearing only some of the costs; other costs are shifted to involuntary creditors. This is a real cost of limited liability, but its magnitude is reduced by corporation's incentives to insure. (see p. 107).

The involuntary creditors are victims of torts, and, as Easterbrook and Fischel note, cannot negotiate with the firm in advance. There is apparently no model in the literature which establishes either the claim that a negative externality[3] exists or that it is reduced due to the firm's incentive to insure. The analysis here shows that, given no change in the investment level, purchasing liability insurance will not create value but will shift value between claimholders and may appear to change value because the financial market value of the corporation does not fully reflect the value of all the stakeholders' claims. This follows because the costs shifted to involuntary creditors are represented, in part, by the value of the stockholders' put option.[4] The incentive to insure is more problematic. The analysis, however, shows that in some cases corporate management has an incentive to purchase liability insurance and that the insurance provides management with the incentive to select efficient investment levels.[5] Equivalently, the corporate liability insurance eliminates the effects of the negative externality. Hence, the analysis here provides a proof of the claims made by Easterbrook and Fischel.

Easterbrook and Fischel also expand the menu of contracts included in the nexus of contracts which defines the corporation.[6] In a similar vein, Cornell

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possible to provide a different interpretation of the put option and that management's incentives may be altered when it has a positive value.

[3] A negative externality exists when the actions of one agent adversely affect those of another outside of the market, e.g. a firm which generates pollution as a byproduct of its production process can adversely affect the environment of other agents. The negative externality exists because of the absence of a contractual relationship between the firm and other agents. If a contractual relationship existed then it could be structured to eliminate the externality, as is shown in the subsequent analysis.

[4] The value of the put option may also be interpreted as the value of limited liability.

[5] The term efficiency is used throughout the article and refers to Pareto efficient allocations.

[6] Coase (1937) provided the insight for this approach. It has been extended by a number of

and Shapiro (1987) expand the set of contracts. Cornell and Shapiro introduce "stakeholders". The group of stakeholders includes not only stockholders and bondholders but also other agents who have explicit or implicit contractual relationships with the corporation. Although Cornell and Shapiro do not explicitly include them, it is apparent that involuntary creditors must be considered to be part of the group of corporate stakeholders. Cornell and Shapiro do claim that the existence of implicit contracts can affect the financial value of the corporation. An equivalent interpretation of their claim is that the existence of implicit contracts generates the potential for negative externalities. A generalization of the Coase Theorem, which includes uncertainty, then suggests that the corporation's contract set can be selected in a way which internalizes the externality and improves the welfare of all parties.[7] It is apparent that this forms the basis for Cornell and Shapiro's claim that the existence of implicit contracts has important implications for corporate finance. The model provided here shows that the existence of involuntary creditors does cause the financial market value of the firm to depend on the composition of its contract set.[8] Therefore, the model provides the basis for establishing Cornell and Shapiro's claims and does establish them for one stakeholder group, i.e., the involuntary creditors.[9]

With regard to the firm's contract set, Mayers and Smith (1982) argue that the corporate form of organization provides investors with an effective hedge because stockholders can eliminate insurable risk through diversification. This argument is used to claim that the value of the insured corporation is the same as the value of the uninsured corporation. In a setting with liability claimants, this claim is true if the probability of insolvency is zero, but it is not if the probability of insolvency is positive. If the insolvency event has a positive probability, then, *ceteris paribus*, the value of the insured firm is less than the value of the uninsured firm. Equivalently, the insurance increases the value of liability claimants' stake in the firm and so reduces the value of the shareholders' limited liability. The elements of the financial market model are presented in the next section and then the value of the financial and non-financial claim holders stakes in the firm are analyzed in the section entitled "Financial Market Values."

Corporate insurance can play a positive role in aligning incentives and in some cases eliminating agency costs. Mayers and Smith (1987) and MacMinn (1987) show that insurance can be used to eliminate or reduce the agency costs due to underinvestment. There is a difference, however, between showing that a contract can solve a problem and showing that the corporate manager has an incentive to use the contract. A corporate manager acting solely in the

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others, including Alchian and Demsetz (1972), Fama (1985), Jensen and Meckling (1976), and Fama and Jensen (1983) and (1985).

[7] See Coase (1960).

[8] If the corporation is viewed as a set of financial contracts, then a generalization of the 1958 Modigliani-Miller Theorem would say that the composition of the contract set is irrelevant.

[9] A extension of this model which allowed for other groups of implicit contract holders would establish Cornell and Shapiro's claims in a more general setting.

interests of stockholders does not, *ceteris paribus*, necessarily have an incentive to purchase liability insurance because it reduces the value of the shareholders' limited liability.[10] Corporate managers, however, make decisions not only on corporate account, but also on personal account. If the corporation's probability of insolvency is zero and the manager is a current shareholder then the manager's decisions on personal and corporate account do not conflict, in this competitive market economy. If, however, the corporation's probability of insolvency is not zero and the manager is a current shareholder then there can be a conflict. In fact, the standard unanimity result can break down.[11]

In the competitive economy analyzed here, firm operations generate liability losses which are absorbed by individual agents if the firm is insolvent. The rationale for the break down in the unanimity result is simply that the manager, *ceteris paribus*, seeks measures to minimize potential liability losses on personal account. The operating decision made by a particular agent depends on the extent of the losses that would have to be absorbed in the event of insolvency.[12] Hence, agents with different loss functions would make different operating decisions. In general, neither the operating decision which maximizes the value of the current shareholders' stake in the firm nor the operating decision of a self interested manager is efficient. The efficient investment level generates a risk adjusted marginal benefit equal to its risk adjusted marginal social cost.[13] Both the incentive and the efficiency characteristics of the liability insurance decision are investigated in the section entitled "Liability Insurance and Corporate Objectives." The final section presents some conclusions and comments on the role which liability insurance plays in managing corporate risk.

The Financial Market Model

Consider an economy with competitive and complete financial markets.[14] Suppose that firms make investment and insurance decisions now and receive

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[10] MacMinn (1987) showed that both the bondholders and stockholders could be made better off by an appropriately structured insurance contract. The contract was designed so that the value of the other stakeholders' claims was not increased by the insurance.

[11] The Unanimity Theorem says that management has the incentive to make decisions on corporate account which are unanimously supported by all shareholders. See DeAngelo (1981), Leland (1974), Ekern and Wilson (1974), and Radner (1974).

[12] The terms, operating decision and investment decisions are used synonymously here.

[13] See MacMinn (1989) for a derivation of the condition for a Pareto efficient investment decision.

[14] Conflict of interest problems are endemic to complete as well as incomplete financial market models, e.g., see MacMinn (1987). One advantage of the complete markets model is that all contract values can be expressed in terms of the basis stock prices since those prices aggregate the investors' risk preferences and probability beliefs. This approach also yields an explicit statement of the objective function which the manager uses for all decisions made on corporate account. MacMinn (1987) shows how the insurance contract is priced in a complete markets setting. That analysis can be generalized to an incomplete markets setting if other financial contracts exist which

random payoffs on the decisions then.[15] Suppose that the operations of the corporation have the potential to harm some or all of the agents in the economy. Also, suppose that the corporation is, *ceteris paribus*, liable for these damages. If there is a positive probability that the corporate earnings do not cover the corporate liabilities, then limited liability protects the firm's shareholders. The event that earnings do not cover liabilities also creates difficulties for the operation of the financial market system due to the potential it creates for the misallocation of resources.

Let (Ω, B) be the measure space where Ω is the set of states of nature and B is the event space. The state space is assumed to be finite. In the complete financial market system, it is possible to construct stock contracts which payoff one dollar in a particular state $\omega \in \Omega$, and zero otherwise. Call these assets the basis stocks. Let the price of basis stock of type ω, be $p(\omega)$. The corporation which has a positive probability of insolvency is treated separately. Let $\Pi_f (I_f, \omega)$ denote the firm's payoff where I_f is the capital value of the input and $\omega \in \Omega$ is the state of nature. Let $L_f (I_f, \omega)$ denote the corporate losses due to liability claims. Let I denote the set of agents i in the economy and let $L_{if} \leq 0$ denote the loss function of agent $i \in I$ due to corporate operations. The total liability of the corporation is

$$L_f = \Sigma_I L_{if}$$

Suppose, for the moment, that the corporation is unlevered. Then the firm is solvent if its earnings cover its liability, i.e., $\Pi_f - L_f \geq 0$. Let $S \in B$ denote the solvency event and let $S^c \in B$ denote the insolvency event.[16] The insolvency event is the relative complement of S with respect to Ω, i.e., $S^c = \Omega \backslash S$ and $S^c = \{\omega \in \Omega \mid \Pi_f - L_f < 0\}$.

Let Ψ_i denote the agent's subjective probability distribution. Each agent has a utility function $u_i: D \rightarrow R$, $D \subset R^2$, which represents preferences for consumption now and then. Consumption now is certain but consumption then depends on the agent's decisions and the state of nature which occurs. The agents determine consumption now and then by purchasing/selling basis and corporate stock. The agent's expected utility is

$$\Sigma_\Omega u_i(c_{i0}, c_{i1}(\omega)) \psi_i(\omega)$$

where the pair $c_i = (c_{i0}, c_{i1})$ represents consumption now and then, respectively. Each agent makes decisions now to maximize expected utility.

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span the payoffs of the insurance contracts. In a more general setting, however, in which spanning conditions are not met, pricing insurance contracts remains an unsolved problem.

[15] There are two dates, "now" and "then". All decisions are made now and all payoffs on those decisions are received then.

[16] In the subsequent sections, where it is important to distinguish between insured and uninsured, the solvency events of the insured and uninsured will be denoted by *I* and *U*, respectively.

In the competitive and complete financial market system, the market value of corporation f is

$$V_f^C = \Sigma_\Omega\, p(\omega) \max\{0, \Pi_f(I_f, \omega) - L_f(I_f, \omega)\}$$
$$= \Sigma_S\, p(\omega)\, [\Pi_f(I_f, \omega) - L_f(I_f, \omega)]^{17}$$

Equivalently, the value of the incorporated firm is the risk adjusted present value of a portfolio of basis stock which has the same payoff structure as the corporation. Note that the corporate value may also be viewed as the value of a portfolio of call options where $L_f(I_f, \omega)$ is the exercise price for the state ω call option. Alternatively, since $\max\{0, \Pi_f - L_f\} = \Pi_f - L_f + \max\{0, L_f - \Pi_f\}$, it follows that corporate value may be equivalently expressed as

$$V_f^C = \Sigma_\Omega\, p(\omega)\, [\Pi_f - L_f] + \Sigma_\Omega\, p(\omega) \max\{0, L_f - \Pi_f\} = V_f^P + P_f$$

where V_f^P ia the value of unincorporated firm[18] and P_f is the value of a portfolio of put options. The exercise price for the state ω put option is $L_f(I_f, \omega)$. A put option will be exercised if the insolvency event occurs, i.e., for $\omega \in S^c$. Due to limited liability, the incorporated firm shareholders can put the firm to the liability claimants. The owners of an unincorporated firm do not have that option. Hence, the value of the incorporated firm exceeds that of the unincorporated firm by the value of the portfolio of put options, i.e., $V_f^C - V_f^P = P_f > 0$, if $P(S^c) > 0$. This divergence of value can, *ceteris paribus*, create resource allocation problems and corporate management problems.

Financial Market Values

The impact of liability losses on corporate value is examined in this section. Liability losses, in part, determine the probability of bankruptcy. Unlike other losses, liability losses do not reduce the corporation's liquidating value. Rather, these losses increase the volume of claims. According to U. S. Bankruptcy law, e.g., see Smith and Robertson 1977, except for the six priorities and secured creditors[19], all other claimants, whether they are debtors, tort

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[17] The C superscript distinguishes this value from that of the unincorporated firm value. The unincorporated firm has a superscript P to denote proprietorship or partnership.

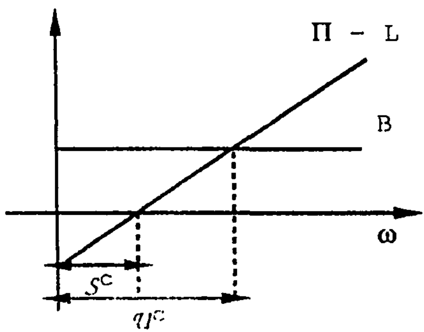
[18] This firm may be a partnership or some other form of organization in which the owners do not have limited liability. This expression does implicitly contain the assumption that the wealth of the partners is sufficient to cover any losses; otherwise, limited liability kicks in again. Alternatively, the value of the proprietorship or partnership, i.e., V_f^P, can be interpreted as an artificial construct. It is used in the subsequent analysis to construct comparisons. It represents a base case in which all losses can be covered.

[19] See Smith and Robertson (1977). Secured creditors with a security interest in the debtor's collateral rank ahead of unsecured claims. Smith and Robertson note that the secured creditor has two courses open to him or her upon the bankruptcy of a debtor: (1) He or she can waive his or her security, prove a claim for the full amount, and participate in the assets on an equal footing with unsecured creditors, or (2) can convert his or her security into money, under the control of the bankruptcy court, credit the amount of such money against the debt, and prove claim for the balance of the debt.

claimants or liability claimants, are treated as general creditors. Even potential tort claimants have been treated as general creditors in an asbestos case, e.g., see Jackson (1986). General creditors share the remaining value of the firm after priority claimants are fully compensated. The distribution is generally done on a pro rata basis, i.e., each creditor receives a proportion of the liquidating value equal to that creditor's proportional ownership of the total liability. Therefore, when the probability of bankruptcy cannot be eliminated by insurance coverages available in the market place, liability insurance becomes relevant to creditors. Liability insurance policies take various forms. For simplicity, it is assumed here that firms purchase a comprehensive liability insurance contract with a specified maximum limit.

The debt, equity and liability claim values are derived first without insurance, and then with it. For simplicity, the corporate payoff is assumed to be an increasing function of ω.[20] The firm issues zero coupon bonds now and pays B dollars then, if the corporate payoff is sufficient. The corporate payoff is shown in figure one.[21] Let U denote the solvency event for the levered uninsured firm.[22] Then $U = \{\omega \in \Omega \mid \Pi_f - L_f \geq B_f\}$. Similarly, let $U^c \equiv \Omega \backslash U$ denote the event that the corporation is insolvent. Bondholders receive B_f

Figure 1
The Insolvency and Bankruptcy Events



[20]The losses are assumed to be increasing in state and $\Pi'(\omega) > L'(\omega) > 0$. Both the payoff Π_f and the losses L_f are functions of the investment as well but that argument is suppressed in this section.

[21]The payoffs are drawn as continuous functions of ω so that the payoffs and corresponding events can be easily conceptualized. The state space is still assumed to be finite. For simplicity, the payoffs are also drawn as linear functions but the analysis does not depend on that representation.

[22]The events U and S are equivalent in the absence of a bond issue. The solvency event of the uninsured all equity firm is specified as U in the next section and compared to the solvency event I of the insured all equity firm.

dollars and stockholders receive $\Pi_f - L_f - B_f$ dollars in the solvency event. In the bankruptcy event U^c, bondholders receive

$$\frac{B_f}{B_f + L_f(\omega)} \Pi_f(\omega)$$

Similarly, the tort claimants, i.e., the other general creditors, receive L_f in the event of no bankruptcy, while they receive

$$\frac{L_f(\omega)}{B_f + L_f(\omega)} \Pi_f(\omega)$$

in the event of bankruptcy. The market value of each set of claims may be determined in this complete financial market system. Let D_f^U, S_f^U and T_f^U denote the uninsured debt, equity and tort values, respectively. The value of the debt is

$$D_f^U = \Sigma_{U^c}\, p(\omega) \frac{B_f}{B_f + L_f(\omega)} \Pi_f(\omega) + \Sigma_U\, p(\omega)\, B_f$$

The stock market value is

$$S_f^U = \Sigma_U\, p(\omega)\, [\Pi_f(\omega) - L_f(\omega) - B_f]$$

Finally, the value of the tort claims is

$$T_f^U = \Sigma_{U^c}\, p(\omega) \frac{L_f(\omega)}{B_f + L_f(\omega)} \Pi_f(\omega) + \Sigma_U\, p(\omega)\, L_f(\omega)$$

The value of the uninsured firm is V_f^U and is the sum of the values of its financial claims, i.e.,

$$V_f^U \equiv D_f^U + S_f^U$$

$$= \Sigma_{U^c}\, p(\omega) \frac{B_f}{B_f + L_f(\omega)} \Pi_f(\omega) + \Sigma_U\, p(\omega)\, B_f$$

$$+ \Sigma_U\, p(\omega)\, [\Pi_f(\omega) - L_f(\omega) - B_f]$$

$$= \Sigma_{U^c}\, p(\omega) \frac{B_f}{B_f + L_f(\omega)} \Pi_f(\omega) + \Sigma_U\, p(\omega)\, [\Pi_f(\omega) - L_f(\omega)]$$

Similarly, the sum of the values of the financial and non-financial claims is

$$V_f^U + T_f^U = \Sigma_{U^c}\, p(\omega) \frac{B_f}{B_f + L_f(\omega)} \Pi_f(\omega) + \Sigma_U\, p(\omega)\, [\Pi_f(\omega) - L_f(\omega)]$$

$$+ \Sigma_{U^c}\, p(\omega) \frac{L_f(\omega)}{B_f + L_f(\omega)} \Pi_f(\omega) + \Sigma_U\, p(\omega)\, L_f(\omega)$$

$$= \Sigma_\Omega\, p(\omega)\, \Pi_f(\omega)$$

and so is reminiscent of the 1958 Modigliani-Miller Theorem. It should be noted that this corporate value V_f^U is a financial market value. It does not directly include the value of liability claims. The distinction between financial market value and total value, i.e., including the liability claims, is an important one because the manager of a publicly held and traded corporation has a fiduciary responsibility to stockholders. If the corporate manager acts in the interests of stockholders and the bondholders' trustee successfully protects the interests of bondholders, then the actions taken by the corporate manager will generally maximize the financial market value of the firm. Some management actions, however, may increase financial market value by reducing the value of the tort claims. Equivalently, some management actions may increase financial market value by increasing the value of the stockholders' limited liability.

Next, consider the value of the firm when liability insurance coverage is purchased. Liability insurance generally covers losses up to a limit k. The analysis here specifies the contract in its generic form. The corporation determines an insurance scheme by selecting an upper limit k for its liability insurance. In a competitive and complete financial market system, competitive insurance premia are offered by insurers. The premia are the risk adjusted present values of the underwriting costs. A liability insurance policy with an upper limit of k dollars pays k if losses are greater than k and pays the loss amount L_f otherwise, i.e., the payoff on the liability insurance is min $\{L_f, k\}$. Let $\mathcal{L}$ denote the event that the losses do not exceed the maximum. Then, as shown in figure two, $\mathcal{L}$ is the event that all liability losses are covered. The insurance premium for such a liability insurance policy is

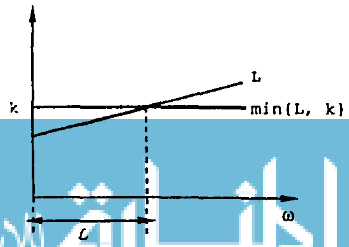
$$p(k) = \Sigma_\Omega\, p(\omega) \min\{L_f(\omega), k\} = \Sigma_{\mathcal{L}}\, p(\omega)\, L_f(\omega) + \Sigma_{\mathcal{L}^c}\, p(\omega)\, k.$$

If min $\{L_f(\omega), k\} = k$ for all $\omega \in \Omega$, then the event $\mathcal{L}^C = \Omega$ and

$$p(k) = \Sigma_\Omega\, p(\omega)\, k = q\, k$$

where q is the sum of the basis stock prices.

Figure 2
Liability Insurance Coverage



The payoff accruing to the insured firm's claim holders are obtained by adding the benefits from insurance coverage to the earnings accruing to the uninsured firm. Let Π_f^U and Π_f^I denote the payoff of the uninsured and insured corporation, respectively. Then

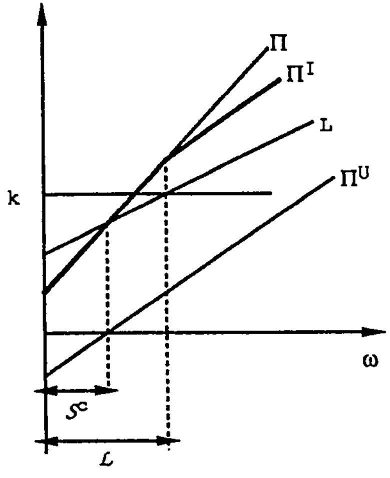
$$\Pi_f^U = \Pi_f - L_f$$

and

$$\Pi_f^I = \Pi_f^U + \min\{k, L_f\}$$

Figure three illustrates the net corporate earnings of the insured and uninsured firms. Since $\Pi_f^I \geq \Pi_f^U$ for all $\omega \in \Omega$, it follows that the insolvency event of the insured firm is a subset of the same event for the uninsured firm. Insurance decisions are made and premia are paid now. Let I denote the solvency event for the insured corporation, i.e., $I \equiv \{\omega \in \Omega \mid \Pi_f^I(\omega) \geq B_f\}$. Then $U \subset I$.

Figure 3
Liabilty Coverage



The payoff to stockholders of the insured firm is max $\{0, \Pi_f^I - B_f\}$, where

$$\max\{0, \Pi_f^I - B_f\} = \begin{cases} 0 & \omega \in I^c \\ \Pi_f - B_f & \omega \in I \cap L \\ \Pi_f - L_f + k - B_f & \omega \in I \cap L^c \end{cases}$$

Note, for example, that $I \cap L$ is the event that the firm is solvent and liability losses are fully covered. Of course, the stockholders pay for the insurance policy and so the stock market value of the insured firm is S_f^I, where

$$
\begin{aligned}
S_f^I &= -p(k) + \Sigma_{I\cap L}\, p(\omega)\, [\Pi_f(\omega) - B_f] \\
&+ \Sigma_{I\cap L^c}\, p(\omega)\, [\Pi_f(\omega) - L_f(\omega) + k - B_f] \\
&= -\Sigma_L\, p(\omega)\, L_f(\omega) - \Sigma_{L^c}\, p(\omega)\, k + \Sigma_{I\cap L}\, p(\omega)\, [\Pi_f(\omega) - B_f] \\
&+ \Sigma_{I\cap L^c}\, p(\omega)\, [\Pi_f(\omega) - L_f(\omega) + k - B_f] \\
&= -\Sigma_{I^c\cap L}\, p(\omega)\, L_f(\omega) - \Sigma_{I^c\cap L^c}\, p(\omega)\, k \\
&+ \Sigma_I p(\omega)\, [\Pi_f(\omega) - L_f(\omega) - B_f]
\end{aligned}
$$

The first two terms represent the portion of the insurance premium which has no corresponding benefit for the shareholders. Other things being equal, a portion of the premium represents a transfer of value from shareholders to general creditors. Equivalently, part of the premium represents a reduction in the value of the limited liability possessed by the shareholders.

Bondholders receive the promised payoff of B_f in the event the firm is solvent. Bondholders share the liquidating payoff of the firm with liability claimants in the event of bankruptcy. The fraction of the payoff received by bondholders in the event of bankruptcy is

$$
\frac{B_f}{B_f + L_f}
$$

Then the payoff to bondholders is

$$
\frac{B_f}{B_f + L_f} \min\{\Pi_f^I, B_f\} = \begin{cases} \dfrac{B_f}{B_f + L_f}\,[\Pi_f + L_f] & \omega \in I^C \cap L \\[2ex] \dfrac{B_f}{B_f + L_f}\,[\Pi_f + k] & \omega \in I^C \cap L^C \\[2ex] B_f & \omega \in I \end{cases}
$$

Hence, the value of the risky bond issue is

$$
\begin{aligned}
D_f^I &= \Sigma_{I^c\cap L}\, p(\omega)\, \frac{B_f}{B_f + L_f(\omega)}\, [\Pi_f(\omega) + L_f(\omega)] \\
&+ \Sigma_{I^c\cap L^c}\, p(\omega)\, \frac{B_f}{B_f + L_f(\omega)}\, [\Pi_f(\omega) + k] \\
&+ \Sigma_I\, p(\omega)\, B_f
\end{aligned}
$$

This representation makes it clear that an increase in the cap on the liability insurance can increase the value of the bond issue when the event L also yields the event I^C. No further increase in k affects the bankruptcy event and so the bond market value is not affected by greater coverage. Similarly, the insurance, *ceteris paribus*, increases the value of the bond issue, as the following proposition shows.

PROPOSITION 1. Given no market imperfections and a strictly positive probability of insolvency, the bond market value of the insured firm is greater than the bond market value of the uninsured firm, i.e., $D_f^I > D_f^U$.

Note that, given liability insurance, the payoff to tort claimants depends on whether the event L or its complement occurs. If the event L occurs then the tort claimants are paid in full; otherwise the tort claimants receive

$$\frac{L_f}{B_f + L_f}$$

of the liquidating payoff Π_f. Hence, the value of the tort claims is

$$T_f^I = \Sigma_{I^c \cap L}\, p(\omega) \frac{L_f(\omega)}{B_f + L_f(\omega)} [\Pi_f(\omega) + L_f(\omega)]$$

$$+ \Sigma_{I^c \cap L^c}\, p(\omega) \frac{L_f(\omega)}{B_f + L_f(\omega)} [\Pi_f(\omega) + k] + \Sigma_I\, p(\omega)\, L_f(\omega)$$

Note that an increase in the cap on the liability insurance increases the probability of the event L and so increases the value of the tort claims.

The financial market value of the insured firm is V_f^I, where

$$V_f^I = D_f^I + S_f^I$$

$$= \Sigma_{I^c \cap L}\, p(\omega) \frac{B_f}{B_f + L_f(\omega)} [\Pi_f(\omega) + L_f(\omega)]$$

$$+ \Sigma_{I^c \cap L^c}\, p(\omega) \frac{B_f}{B_f + L_f(\omega)} [\Pi_f(\omega) + k]$$

$$+ \Sigma_I\, p(\omega)\, B_f - \Sigma_L\, p(\omega)\, L_f(\omega) - \Sigma_{L^c}\, p(\omega)\, k$$

$$+ \Sigma_{I \cap L}\, p(\omega)\, [\Pi_f(\omega) - B_f]$$

$$+ \Sigma_{I \cap L^c}\, p(\omega)\, [\Pi_f(\omega) - L_f(\omega) + k - b_f]$$

$$= \Sigma_{I^c}\, p(\omega) \frac{B_f}{B_f + L_f(\omega)} \Pi_f(\omega) + \Sigma_I\, p(\omega)\, [\Pi_f(\omega) - L_f(\omega)]$$

Similarly, the sum of the financial and non-financial claims is

$$V_f^I + T_f^I = \Sigma_\Omega\, p(\omega)\, \Pi_f(\omega)$$

This analysis is summarized in the following propositions:

PROPOSITION 2. In the absence of market imperfections, the sum of the values of the financial and non-financial claims is the same whether the firm is insured or not, i.e., $V_f^I + T_f^I = V_f^U + T_f^U$.

PROPOSITION 3. Given no market imperfections and a strictly positive probability of insolvency, the financial market value of the insured firm is less than the financial market value of the uninsured firm, i.e., $V_f^I < V_f^U$.

Proposition two shows that, *ceteris paribus,* insuring the firm's liabilities does not create value but it does shift value between the different groups of claim holders. In particular, propositions one and three show that bondholders and tort claimants benefit from the insurance. It follows, then, that shareholders are always worse off with insurance, because they pay the insurance premium only to lose the value provided by limited liability. This implies that any corporate management which acts strictly in the interests of current shareholders will not choose to insure.

Liability Insurance and Corporate Objectives

Using the Fisher model in this complete financial market setting and letting the corporate manager make the firm's investment decision on corporate account as well as a portfolio decision on personal account, it is possible to generate an objective function which the manager uses in making the corporate decisions. This objective function shows that a unanimity result does not generally hold, and that the manager's investment choice is not efficient. The efficient investment maximizes the risk adjusted net present value $V_f^{UP} - I_f$. Recall that V_f^{UP} is the value of an uninsured proprietorship or partnership which is, therefore, subject to unlimited liability. It follows that the investment choice which maximizes the risk adjusted net present value of the partnership internalizes all of the costs associated with the operation of the firm. The manager of the publicly held and traded corporation, however, faces the possibility of losses on personal account in the event of corporate insolvency. The purpose of this section is to characterize and compare the investment decisions of the manager with and without liability insurance.

Suppose agent i is the manager of corporation f. Also suppose that the manager has an initial endowment of stock in corporation f. Let (m_{i0}, m_{i1}) denote the income pair of agent i now and then, respectively.[23] Let $x_{if}^0 > 0$ denote the number of shares of common stock initially held by manager i and let x_{if} denote the number of shares held after trading now.[24] Suppose the manager makes the investment decision for the firm now and uses a new stock issue to finance the investment.[25] Let S_f^N denote the value of the new stock issue and let I_f denote the dollar investment. Suppose the firm has issued N_f shares of stock previously and issues n_f new shares to finance the investment of I_f dollars. With no liability insurance, the manager's consumption pair may be expressed as

$$c_{i0} = m_{i0} - \Sigma_\Omega\, p(\omega)\, x_i(\omega) + p_f(x_{if}^0 - x_{if})$$

[23] The analysis here abstracts from the operation of product and factor markets. The income pair noted here is due to the operation of those markets.

[24] This type of assumption generally makes the manager's decisions consistent with the interests of stockholders and so also generally provides a Fisher Separation result. One could also ask what type of a compensation scheme would provide the manager with an incentive to select the efficient investment level.

[25] The analysis could be altered to allow for a bond issue rather than a stock issue.

$$c_{i1}(\omega) = \begin{cases} m_{i1} + x_i(\omega) + \dfrac{x_{if}}{N_f + n_f}[\Pi_f(I_f, \omega) - L_f(I_f, \omega)] & \text{for } \omega \in U \\ m_{i1} + x_i(\omega) - L_{if}(I_f, \omega) + \dfrac{L_{if}(I_f, \omega)}{L_f(I_f, \omega)}\Pi_f(I_f, \omega) & \text{for } \omega \in U^C \end{cases}$$

where $x_i(\omega)$ is the number of shares of basis stock the manager holds after trading.[26] Notice that agent i does have a loss $L_{if}(\omega)$ for $\omega \in U$ but it is fully covered by the corporation. In the insolvency event U^C the agent's loss, other things being equal, is not fully covered. In this environment, the manager has contradicting interests in the corporation. As a stockholder, the manager has an incentive to maximize the value of the current shareholders' stake in the firm. As an employee and a potential claimant, the manager has an incentive to protect personal wealth. Hence, the manager must resolve these conflicting interests when making decisions on corporate account. Given competitive and complete financial markets, it has been shown that the manager resolves these conflicting interests by maximizing a weighted average of the current shareholder value and the risk-adjusted present value of the manager's wealth loss due to insolvency.[27] The objective function is

$$\alpha_{if} S_f^{U0} + W_{if}^U$$

where S_f^{U0} is the uninsured stock value of the old shareholders' stake in the firm, W_{if}^U is the risk adjusted present value of the manager's wealth loss, and α_{if} is the manager's initial ownership stake in the corporation.[28] Alternatively, let β_{if} denote the fractional liability claim of the manager, i.e.,

$$\beta_{if}(I_f, \omega) = \frac{L_{if}(I_f, \omega)}{L_f(L_f, \omega)}$$

Then, the decisions made by the manager will depend on the effect that the investment decision has on both the market value of the corporation and the manager's claim in the event of corporate insolvency. For simplicity, it is assumed here that

$$\frac{D_1 L_{if}}{L_{if}} = \frac{D_1 L_f}{L_f} \text{ and } \frac{D_2 L_{if}}{L_{if}} = \frac{D_2 L_f}{L_f}$$ [29]

Then it follows that the manager's proportional claim in the event of insolvency is independent of both the investment level and the state of nature,

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[26] The representation of $c_{i1}(\omega)$ implicitly assumes that income then, i.e., m_{i1}, is large enough so that consumption then is non-negative, despite the losses in the insolvency state. Without this assumption it would be necessary to consider limited liability on personal as well as corporate account.

[27] See MacMinn (1989) for a derivation of this objective function.

[28] In terms of shares of common stock, $\alpha_{if} = x_{if}^0/N_f$.

[29] The notation D_1L_f and D_2L_f denotes the partial derivatives of the function L_f with respect to the first and second arguments, respectively.

i.e., $D_1\beta_{if}(I_f, \omega) = D_2\beta_{if}(I_f, \omega) = 0$. It follows that the manager's wealth loss can be rewritten as follows

$$W_{if}^U = \Sigma_{\mathcal{U}^c}\, p(\omega)\, [-L_{if} + \frac{L_{if}}{L_f}\Pi_f]$$

$$= \beta_{if}\, \Sigma_{\mathcal{U}^c}\, p(\omega)\, [\Pi_f - L_f]$$

$$= \beta_{if}\, W_f^U$$

where W_f^U represents the aggregate wealth loss of liability claimants. This allows the manager's objective function to be rewritten as

$$\alpha_{if}\, S_f^{UO} + \beta_{if}\, W_f^U \quad (1)$$

or equivalently, as

$$\alpha_{if}\,(S_f^{UP} - I_f) + (\beta_{if} - \alpha_{if})\, W_f^U \quad (2)$$ [30]

From (1) it is clear that the manager acts in the interests of current shareholders if $\beta_{if} = 0$ and from (2) it is clear that acting in the interests of current shareholders does not result in an efficient investment level.

Consider how the manager's investment decision compares to the efficient investment level. Recall that the efficient investment level I_f^E is implicitly defined by the condition

$$\alpha_{if}\left(\frac{dS_f^{UP}}{dI_f} - 1\right) = \alpha_{if}\left(\Sigma_\Omega\, p(\omega)\, [D_1\Pi_f - D_1L_f] - 1\right) = 0$$

Assume that the increase in the firm's payoff exceeds the increase in the firm's liability as the investment level increases and that the marginal payoffs and liabilities increase at decreasing rates, i.e., $D_1\Pi_f - D_1L_f > 0$ and that $D_{11}\Pi_f - D_{11}L_f < 0$ for all I_f and $\omega \in \Omega$. These assumptions imply that a larger investment reduces the probability of insolvency without necessarily eliminating it. These assumptions also yield an aggregate wealth loss function W_f^U which is increasing and concave in I_f. Clearly, if the manager has a proportional ownership of the corporation equal to the proportional losses in the event of insolvency then the efficient investment level will be selected. Otherwise, the investment choice depends on whether the additional investment benefits the manager more as a stockholder or as a liability claimant. The manager's condition for an optimal investment level is

$$\alpha_{if}\frac{dS_f^{UO}}{dI_f} + \beta_{if}\frac{dW_f^U}{dI_f} =$$

[30] To see this, note that $S_f^U = S_f^{UO} + S_f^{UN}$ and $S_f^{UN} = I_f$. It follows that

$$\alpha_{if}(\Sigma_{\mathcal{U}}\, p(\omega)\, [\Pi_f(I_f, \omega) - L_f(I_f, \omega)] - I_f) + \Sigma_{\mathcal{U}^c}\, p(\omega)\, [-L_{if} + \frac{L_{if}}{L_f}\Pi_f] =$$

$$\alpha_{if}(\Sigma_\Omega\, p(\omega)\, [\Pi_f - L_f] - I_f) + (\beta_{if} - \alpha_{if})\, \Sigma_{\mathcal{U}^c}\, p(\omega)\, [\Pi_f - L_f]$$

$$\alpha_{if}\left(\frac{dS_f^{UP}}{dI_f}-1\right)+(\beta_{if}-\alpha_{if})\frac{dW_f^U}{dI_f} = 0 \quad (3)$$

Consider the manager whose percentage loss in the event of insolvency is less than his or her initial percentage ownership of the firm, i.e., $\beta_{if} < \alpha_{if}$. This manager selects an investment level less than the efficient level. The converse is true, if the manager's percentage loss is greater than his or her percentage ownership. The rationale is that if the manager initially has a 5 percent stake in the firm, then he or she shares 5 percent of the benefits and 5 percent of the costs. He or she will share 5 percent of the profits due to an increase in the investment if the firm remains solvent. However, if the firm subsequently becomes insolvent then the investment would benefit liability claimants and thereby reduce the value of the put option that stockholders have. If the manager assesses that his or her percentage liability claim is 3 percent, then he or she is essentially paying 5 percent of the costs as a stockholder and gaining 3 percent of the benefits as a liability claimant. It becomes apparent that he or she will not push investment to the efficient level. Conversely, if the manager holds 3 percent of the firm's stocks and 5 percent of the liability claims, then he or she will receive 5 percent of the benefit from investment and pay 3 percent of the costs in the event of insolvency. Hence, he or she has an incentive to push investment beyond the efficient level. Let I_f^S denote the investment level which maximizes the current shareholders' value and let I_f^M denote the investment level selected by the manager. Then the following proposition summarizes these results.

PROPOSITION 4. Given $P\{U^C\} > 0$, the manager selects I_f^M such that $I_f^E > I_f^M > I_f^S$ if $\alpha_{if} > \beta_{if} > 0$ and $I_f^M > I_f^E > I_f^S$ if $\beta_{if} > \alpha_{if} > 0$.

This proposition shows that the manager may either under- or over-invest relative to the efficient investment level. Of course, the efficient investment level is greater than the investment level which maximizes the current shareholders' stake as long as the probability of insolvency is positive.

Next, suppose the manager can purchase liability insurance on corporate account. Since there is no unanimity on the investment decision, it is also to be expected that management and stockholders will disagree on the level of insurance coverage. Since the insurance increases the value of the liability claimants' position while decreasing the value of the equity, there could only be agreement if $\beta_{if} = \beta_{jf}$ for all investors $i, j \in I$.

Recall that the corporate payoff of the insured firm is $\Pi_f^I = \Pi_f^U + \min\{L_f, k\}$ and, of course, the insolvency event is a function of the level of insurance coverage. The insolvency event is $I^C = \{\omega \in \Omega \mid \Pi_f^I(\omega) < 0\}$. Similarly, in the absence of a bond issue, the payoff to stockholders of the insured firm is $\max\{0, \Pi_f^I\}$, where

$$\max\{0, \Pi_f^I\} = \begin{cases} 0 & \omega \in I^C \\ \Pi_f & \omega \in I \cap L \\ \Pi_f - L_f + k & \omega \in I \cap L^C \end{cases}$$

If the cap k on the liability insurance is sufficiently small then the insolvency event has a positive probability, i.e., $P\{I^C\} > 0$. Increasing the cap on the liability insurance will, of course, reduce the probability of insolvency. In this competitive complete market setting, the self interested manager selects the corporate investment level and liability insurance contract to maximize expected utility. The following proposition shows that maximizing expected utility and maximizing an appropriate weighted average of current shareholder value and wealth losses provide equivalent results.

PROPOSITION 5. Suppose a new equity issue is used to finance the corporation's investment and liability insurance. Then selecting the pair (I_f, k) to maximize expected utility is equivalent to selecting the pair to maximize the objective function

$$\alpha_{if}\, S_f^{IO} + \beta_{if}\, W_f^I \tag{4}$$

or the equivalent objective function

$$\alpha_{if}\, (S_f^{IP} - I_f) + (\beta_{if} - \alpha_{if})\, W_f^I \tag{5}$$

The classic Unanimity Theorem states that the self interested corporate manager makes decisions that are unanimously supported.[31] Proposition five shows that managers, with different liability claims, have incentives to make different decisions and that the decisions will not generally be supported by other investors. As long as there is a positive probability of insolvency and the manager has a liability claim, the manager does not have an incentive to act strictly in the interests of the current shareholders.

The objective function, i.e., (5), in proposition five does show that there are conditions which will motivate the corporate manager to purchase liability insurance and that the insurance decision has an effect on the investment decision. The classic result on the demand for corporate insurance is that it will neither increase nor decrease corporate value and so it is a matter of indifference to the manager.[32] The classic result, however, does not allow for a positive probability of insolvency in the absence of a bond issue. The stock market value of an insured versus uninsured firm would be essentially the same as the value of the insured versus uninsured proprietorships in this model. The stock market value of the insured proprietorship is equal to the stock market value of the uninsured proprietorship, i.e., $S_f^{IP} = S_f^{UP}$, since

$$\begin{aligned} S_f^{IP} &= -q\,k + \Sigma_{\Omega}\, p(\omega)\, [\Pi_f(I_f,\omega) - L_f,\, \omega) + k] \\ &= -q\,k + \Sigma_{\Omega}\, p(\omega)\, [\Pi_f(I_f,\, \omega) - L_f(I_f,\, \omega)] + q\,k \\ &= S_f^{UP}. \end{aligned} \tag{6}$$

[31] See DeAngelo (1981). It should be noted that the assumptions of the DeAngelo model preclude the existence of any externalities.

[32] For example, see Mayers and Smith (1982).

Note that using (6), it is also possible to state the manager's objective function as

$$\alpha_{if}\,(S_f^{UP} - I_f) + (\beta_{if} - \alpha_{if})\,W_f^I \qquad (7)$$

This form of the objective function makes it clear that the demand for liability insurance depends on the manager's relative stake in the firm.

When there is a positive probability of insolvency and the firm is a corporation rather than a proprietorship or partnership, the value of the insured corporation is less than that of the uninsured corporation.[33] A stockholder who does not have a liability claim against the firm loses when the firm purchases liability coverage. That stockholder pays, albeit indirectly, a share of the insurance premium but the gain does not cover the expense. If the firm becomes insolvent, then the benefit of insurance all goes to liability claimants. The manager who is both a stockholder and a liability claimant has a different view. The manager receives some benefit from the insurance coverage in the event of insolvency. If the manager's percentage liability claim is higher than his or her percentage ownership of the firm then his or her wealth in the firm will increase with more states being covered at every investment level. This provides the manager with an incentive to purchase coverage for every initially insolvent state. An immediate consequence is that it becomes optimal for the manager to select the efficient investment level. If his or her percentage liability claim is less than his or her percentage ownership of the firm then his or her wealth will decline with insurance coverage at every investment level. In this case, insurance will not be purchased. The rationale is again the balance between costs and benefits. Suppose the manager owns 3 percent of the firm and 5 percent of the liability claims. When the firm purchases insurance, he or she pays 3 percent of the premium. In the event of insolvency, the manager receives 3 percent of the insurance benefits. Recall that the premium is simply the risk-adjusted present value of the potential benefit. From an *ex ante* point of view, the present value of the manager's benefits, i.e., the liability claim which will not be fully covered in the absence of insurance, outweighs the present value of the cost, i.e., the insurance premium. If the situation is reversed, then his or her costs will outweigh his or her benefits and the insurance will not be purchased.

If the manager purchases enough insurance to eliminate the insolvency event then the objective function makes it clear that the manager will select the efficient investment level. To see this, note that differentiating (7) yields the manager's conditions for optimal investment and insurance levels. The derivatives with respect to I_f and k are $\alpha_{if}\,(D_1S_f^{UP} - 1) + (\beta_{if} - \alpha_{if})\,D_1W_f^I = 0$ and $(\beta_{if} - \alpha_{if})\,D_2W_f^I = 0$, respectively.[34] Since $D_2W_f^I > 0$ when $P\{I^C\}$ 0 and $D_2W_f^I = 0$ when $P\{I^C\} = 0$, it is apparent that the manager has an incentive to

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[33] For an example which allows for a positive probability of insolvency see MacMinn (1987).

[34] Since $W_f^I(I_f, k) = \Sigma_{I^C}\, p\,[\Pi_f(I_f, \omega) - L_f(I_f, \omega) + k]$, it follows that $D_1W_f^I = \Sigma_{I^C}\, p(\omega)\,[D_1\Pi_f(I_f, \omega) - D_1L_f(I, \omega)] > 0$ and $D_2W_f^I = \Sigma_{I^C}\, p(\omega) > 0$, for all (I_f, k) such that the insolvency set I^C is not empty.

purchase insurance if $\beta_{if} > \alpha_{if}$. Just as clearly, the manager has no incentive if $\beta_{if} < \alpha_{if}$. Similarly, since the efficient investment satisfies the condition $D_1S_f^{UP} - 1 = 0$, $D_1W_f^I > 0$ when $P\{I^c\} > 0$ and zero otherwise, it is also clear that the manager has an incentive to over-invest if $\beta_{if} > \alpha_{if}$ and under-invest if $\beta_{if} < \alpha_{if}$. The following proposition summarizes these results.

PROPOSITION 6. If $\beta_{if} > \alpha_{if}$, then the manager selects k so that $P\{I^c\} = 0$ and I_f^M such that $I_f^M = I_f^E$. If $\beta_{if} < \alpha_{if}$, then the manager selects $k = 0$ and I_f^M such that $I_f^M < I_f^E$.

This proposition shows that insurance can be important in aligning the interests of management not with the shareholders but with all stakeholders. Therefore, insurance may play a positive role in generating an efficient allocation of resources in a financial market economy characterized by risky business.

Concluding Remarks

The analysis shows that, other things being equal, insurance increases the value of debt and liability claims while reducing the value of equity claims. What is more, as long as there is a positive probability of corporate insolvency, the insurance reduces the financial market value of the corporation because the liability claims are not fully represented in the financial market value.

The role of the corporate manager in making investment and insurance decisions is considered. The manager of a publicly traded corporation that has a positive probability of insolvency does not generally have the incentive to make the socially efficient investment decision. The analysis shows that as a stockholder and a potential liability claimant, the manager weighs his or her roles as stockholder and liability claimant in making investment decision for the firm. When the role as stockholder outweighs the role as liability claimant, the manager's investment decision will be closer but still divergent from the one that maximizes the equity value. If the role as liability claimant outweighs the role as stockholder, then the self interested manger has an incentive to purchase liability insurance. If the manager can eliminate the possibility of insolvency then the manager also has an incentive to make the efficient investment decision.[35]

This analysis has not allowed for anything more than the simplest type of compensation scheme. Managers usually have a substantial portion of compensation tied to the firm's payoff. The manager receives the full amount of compensation only if the firm remains solvent. In the event of insolvency, the manager's claim over regular salary may be considered a priority claim but the claim over other types of compensation may, at best, be considered as another liability claim.[36] Therefore, the manager may have an even stronger

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[35] This statement is based on the assumption that the corporate payoff Π_f is positive for all $\omega \in \Omega$.

[36] There is a cap on the amount that can be considered a priority claim. See Cohen (1981).

incentive to purchase insurance. Propositions four and six imply that, *ceteris paribus*, the manager with a larger net general stake has a bigger incentive to either increase investment or insurance coverage. This is a potentially testable claim but it must be tempered by the recognition that there are other contracting means of reducing the probability of insolvency. The probability of insolvency can also be reduced by hedging in financial futures, e.g., see Smith and Stulz (1985). Further work is necessary to identify the other determinants of the demand for liability insurance and to distinguish the conditions under which the insurance contract dominates other financial contracts.

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Appendix

Proof of Proposition 1:

$$D_f^I - D_f^U = \Sigma_{I^c}\, p(\omega) \frac{B_f}{B_f + L_f(\omega)} [\Pi_f(\omega) + \min\{L_f(\omega), k\}] + \Sigma_I p(\omega)\, B_f$$

$$- \Sigma_{U^c}\, p(\omega) \frac{B_f}{B_f + L_f(\omega)} \Pi_f(\omega) - \Sigma_U\, p(\omega)\, B_f$$

$$= \Sigma_{I \setminus U}\, p(\omega)\, B_f + \Sigma_{I^c}\, p(\omega) \frac{B_f}{B_f + L_f(\omega)} \min\{L_f(\omega), k\}$$

$$- \Sigma_{U^c \setminus I^c}\, p(\omega) \frac{B_f}{B_f + L_f(\omega)} \Pi_f(\omega)$$

$$= \Sigma_{I \setminus U}\, p(\omega)\, [B_f - \frac{B_f}{B_f + L_f(\omega)} \Pi_f(\omega)] + \Sigma_{I^c}\, p(\omega) \frac{B_f}{B_f + L_f(\omega)} \min\{L_f(\omega), k\}$$

$$> 0$$

if

$$B_f - \frac{B_f}{B_f + L_f(\omega)} \Pi_f(\omega) > 0$$

$$\Leftrightarrow B_f\, [1 - \frac{\Pi_f(\omega)}{B_f + L_f(\omega)}] > 0$$

$$\Leftrightarrow 1 > \frac{\Pi_f(\omega)}{B_f + L_f(\omega)}$$

$$\Leftrightarrow B_f + L_f(\omega) > \Pi_f(\omega)$$

The last inequality obviously holds for all $\omega \in I \setminus U = U^C \setminus I^C$. Q.E.D.

Proof of Proposition 3: By proposition two,

$$V^I - V^U = T^U - T^I$$

$$= \Sigma_{U^c}\, p(\omega) \frac{L_f(\omega)}{B_f + L_f(\omega)} \Pi_f(\omega) + \Sigma_U\, p(\omega)\, L_f(\omega)$$

$$- \Sigma_{I^c \cap L}\, p(\omega) \frac{L_f(\omega)}{B_f + L_f(\omega)} [\Pi_f(\omega) + L_f(\omega)]$$

$$- \Sigma_{I^c \cap L^c}\, p(\omega) \frac{L_f(\omega)}{B_f + L_f(\omega)} [\Pi_f(\omega) + k] - \Sigma_I\, p(\omega)\, L_f(\omega)$$

$$= \Sigma_{U^c}\, p(\omega) \frac{L_f(\omega)}{B_f + L_f(\omega)} \Pi_f(\omega) + \Sigma_U\, p(\omega)\, L_f(\omega)$$

$$- \Sigma_{\mathcal{I}^c} p(\omega) \frac{L_f(\omega)}{B_f + L_f(\omega)} [\Pi_f(\omega) + \min\{L_f(\omega), k\}]$$

$$- \Sigma_{\mathcal{I}} p(\omega) L_f(\omega)$$

$$= \Sigma_{\mathcal{U}^c \setminus \mathcal{I}^c} p(\omega) \frac{L_f(\omega)}{B_f + L_f(\omega)} \Pi_f(\omega) - \Sigma_{\mathcal{I}^c} p(\omega) \frac{L_f(\omega)}{B_f + L_f(\omega)} \min\{L_f(\omega), k\}$$

$$- \Sigma_{\mathcal{I} \setminus \mathcal{U}} p(\omega) L_f(\omega)$$

$$= \Sigma_{\mathcal{U}^c \setminus \mathcal{I}^c} p(\omega) \left[\frac{L_f(\omega)}{B_f + L_f(\omega)} \Pi_f(\omega) - L_f(\omega)\right]$$

$$- \Sigma_{\mathcal{I}^c} p(\omega) \frac{L_f(\omega)}{B_f + L_f(\omega)} \min\{L_f(\omega), k\}$$ [37]

$$< 0.$$ [38]

Q.E.D.

Proof of Proposition 5. Suppose the cap on the liability insurance is small so that $P\{\mathcal{I}^C\} > 0$. Then the event that all liability losses are covered is empty. Equivalently $P\{\mathcal{L}\} = 0$. Then the manager's consumption pair is

$$c_{i0} = m_{i0} - \Sigma_{\Omega} p(\omega) x_i(\omega) + p_f(x_{if}{}^0 - x_{if})$$

$$c_{i1}(\omega) = \begin{cases} m_{i1} + x_i(\omega) + \dfrac{x_{if}}{N_f + n_f} [\Pi_f(I_f, \omega) - L(I_f, \omega) + k] & \omega \in \mathcal{I} \cap \mathcal{L}^C \\ m_{i1} + x_i(\omega) - L_{if}(I_f, \omega) + \dfrac{L_{if}(I_f, \omega)}{L_f(I_f, \omega)} [\Pi_f(I_f, \omega) + k] & \omega \in \mathcal{I}^C \cap \mathcal{L}^C \end{cases}$$

In this competitive complete market setting, the self interested manager selects the corporate investment level to maximize expected utility. The manager's expected utility is $\Sigma_{\Omega} u_i (c_{i0}, c_{i1}(\omega)) \psi(\omega)$. The first order conditions for basis stock ζ and corporate stock, i.e., $x_i(\zeta)$ and x_{if}, are

$$-p(\zeta)\Sigma_{\Omega} D_1 u_i \psi(\omega) + D_2 u_i \psi(\zeta) = 0, \zeta \in \Omega \quad \text{(A.1)}$$

$$-p_f \Sigma_{\Omega} D_1 u_i \psi + \frac{1}{N_f + n_f} \Sigma_{\mathcal{I} \cap \mathcal{L}^c} D_2 u_i [\Pi_f - L_f + k] \psi = 0 \quad \text{(A.2)}$$

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[37] This equality follows because $\mathcal{U} \setminus \mathcal{I}$ and $\mathcal{I}^c \setminus \mathcal{U}^c$ are the same.

[38] The inequality follows because

$$\frac{L_f(\omega)}{B_f + L_f(\omega)} \Pi_f(\omega) - L_f(\omega) < 0$$

$$\Leftrightarrow L_f(\omega) \left[\frac{\Pi_f(\omega)}{B_f + L_f(\omega)} - 1\right] < 0$$

$$\Leftrightarrow \Pi_f(\omega) < B_f + L_f(\omega)$$

and the last inequality obviously holds for all $\omega \in \mathcal{U}^C \setminus \mathcal{I}^C$.

respectively. (A.1) yields

$$p(\zeta) = \frac{D_2 u_i \, \psi(\zeta)}{\Sigma_\Omega D_1 u_i \, \psi(\omega)}, \; \zeta \in \Omega \tag{A.3}$$

which is simply the condition that the basis stock price equal the marginal rate of substitution. Similarly, (A.2) yields

$$p_f(I_f, k) = \frac{1}{N_f + n_f} \Sigma_{I \cap L^c} \, p(\omega) \, [\Pi_f - L_f + k]$$

and the stock market value of this insured firm may be written as

$$S_f^I(I_f, k) \equiv p_f \, (N_f + n_f) = \Sigma_{I \cap L^c} \, p(\omega) \, [\Pi_f - L_f + k]$$

$$= \Sigma_I \, p(\omega) \, [\Pi_f - L_f + k]$$

The last equality follows because $I \cap L^C = I$.

Next, consider the investment decision and the manner in which it is financed. Suppose that the manager issues new equity to cover the investment expenditure and the liability insurance. Then

$$p_f \, n_f \equiv S_f^N = \frac{n_f}{N_f + n_f} S_f = I_f + p(k)$$

implicitly defines the conditions for the new issue. Direct calculation show that

$$n_f(I_f, k) = N_f \frac{I_f}{S_f - I_f - p(k)}$$

Since $\min(L_f, k) = k$, it may be noted that

$$p(k) = \Sigma_\Omega \, p(\omega) \min\{L_f(I_f, \omega), k\} = q \, k$$

where q is the sum of the basis stock prices. Then

$$n_f(I_f, k) = N_f \frac{I_f}{S_f - I_f - q \, k}$$

Now, consider the manager's FOC for I_f. Differentiating the manager's expected utility with respect to I_f yields

$$\Sigma_\Omega D_1 u_i \, D_1 p_f \, (x_{if}^0 - x_{if}) \, \psi$$

$$+ \Sigma_I D_2 u_i \left\{ \frac{x_{if}}{N_f + n_f} (D_1 \Pi_f - D_1 L_f) - \frac{x_{if}}{N_f + n_f} \frac{n_f'}{N_f + n_f} (\Pi_f - L_f + k) \right\} \psi$$

$$+ \Sigma_{I^c} D_2 u_i \{ (-D_1 L_{if}) + \frac{L_{if}}{L_f} D_1 \Pi_f + \frac{L_f \, D_f L_{if} - L_{if} \, D_1 L_f}{L_f^2} (\Pi_f + k) \} \, \psi = 0$$

Using (A.3) and the derivative properties of β_{if}, this FOC may be equivalently expressed as

$$D_1 p_f (x_{if}^0 - x_{if})$$

$$+ \Sigma_I\, p(\omega) \left\{ \frac{x_{if}}{N_f + n_f} (D_1\Pi_f - D_1 L_f) - \frac{x_{if}}{N_f + n_f} \frac{n_f'}{N_f + n_f} (\Pi_f - L_f + k) \right\}$$

$$+ \beta_{if} \Sigma_{I^c}\, p(\omega)\, \{D_1\Pi_f - D_1 L_f\} =$$

$$D_1 p_f\, x_{if}^0 + \beta_{if}\, \Sigma_F\, p(\omega)\, \{D_1\Pi_f - D_1 L_f\} = 0 \tag{A.4}$$

Claim 1: $D_1 p_f(I_f, k)\, x_{if}^0 = \alpha_{if}\, D_1 S_f^{I0}(I_f, k)$

Proof of Claim 1. Direct calculation yields

$$D_1 p_f = \frac{1}{N_f + n_f} D_1 S_f^I - \frac{D_1 n_f}{(N_f + n_f)^2} S_f^I$$

$$= \frac{1}{N_f + n_f} \left(D_1 S_f^I - \frac{D_1 n_f}{(N_f + n_f)^2} S_f^I \right)$$

$$= \frac{1}{N_f + n_f} \left(D_1 S_f^I - \frac{N_f}{N_f + n_f} S_f^I \frac{S_f^{IO} - I_f(D_1 S_f^I - 1)}{(S_f^{IO})^2} \right)$$

$$= \frac{1}{N_f + n_f} \left(D_1 S_f^I - S_f^{I0} \frac{S_f^{IO} - I_f(D_1 S_f^I - 1)}{(S_f^{IO})^2} \right)$$

$$= \frac{1}{N_f + n_f} \left(D_1 S_f^I - \frac{S_f^{IO} - I_f(D_1 S_f^I - 1)}{(S_f^{IO})} \right)$$

$$= \frac{1}{N_f + n_f} \left((D_1 S_f^I - 1) \left(1 + \frac{I_f}{(S_f^{IO})} \right) \right)$$

Next, since

$$S_f^{IO} = \frac{N_f}{N_f + n_f} S_f^I$$

Direct calculation shows that

$$D_1 S_f^{IO} = \frac{N_f}{N_f + n_f} D_1 S_f^I - \frac{N_f\, D_1 n_f}{(N_f + n_f)^2} S_f^I$$

$$= \frac{N_f}{N_f + n_f} \left(D_1 S_f^I - \frac{D_1 n_f}{(N_f + n_f)} S_f^I \right)$$

$$= \frac{N_f}{N_f + n_f} \left((D_1 S_f^I - 1) \left(1 + \frac{I_f}{S_f^{I0}} \right) \right)$$

If follows that

$$D_1 p_f\, x_{if}^0 = \frac{x_{if}^0}{N_f + n_f}\left((D_1 S_f^I - 1)\left(1 + \frac{I_f}{S_f^{IO}}\right)\right) = \alpha_{if}\, D_1 S_f^{IO}$$

Q.E.D.

Since claim one holds, it follows that the manager selects the investment level for the corporation by maximizing the following objective function

$$\alpha_{if}\, S_f^{IO} + \beta_{if}\, W_f^I$$

where

$$S_f^{IO} = \frac{N_f}{N_f + n_f} \Sigma_I\, p(\omega)\, [\Pi_f(I_f, \omega) - L_f(I_f, \omega) + k]$$

and

$$W_f^I = \Sigma_{I^c}\, p(\omega)\, [\Pi_f(I_f, \omega) - L_f(I_f, \omega) + k].$$

This objective function may be equivalently expressed as

$$\alpha_{if}\, (S_f^{IP} - I_f) + (\beta_{if} - \alpha_{if})\, W_f^I$$

where S_f^{IP} is the insured value of the proprietorship or partnership and

$$S_f^{IP} = -q\, k + \Sigma_\Omega\, p(\omega)\, [\Pi_f(I_f, \omega) - L_f(I_f, \omega) + k]$$

Finally, consider the manager's choice of k. Differentiating the manager's expected utility with respect to k yields

$$\Sigma_\Omega\, D_1 u_i\, D_2 p_f\, (x_{if}^0 - x_{if})\, \psi$$

$$+ \Sigma_I\, D_2 u_i \left[\frac{x_{if}}{N_f + n_f} - \frac{x_{if}\, D_2 n_f}{(N_f + n_f)^2} (\Pi_f(I_f, \omega) - (L_f, \omega) + k)\right] \psi$$

$$+ \Sigma_{I^c}\, D_2 u_i\, \frac{L_{if}(L_{if}, \omega)}{L_f(L_f, \omega)}\, \psi$$

Multiplying the derivative by one over the expected marginal utility of consumption now yields

$$D_2 p_f\, (x_{if}^0 - x_{if})$$

$$+ \Sigma_I\, p(\omega) \left[\frac{x_{if}}{N_f + n_f} - \frac{x_{if}\, D_2 n_f}{(N_f + n_f)^2} (\Pi_f(I_f, \omega) - L_f(I_f, \omega) + k)\right]$$

$$+ \Sigma_{I^c}\, p(\omega)\, \frac{L_{if}(I_f, \omega)}{L_f(I_f, \omega)}$$

$$= D_2 p_f \, x_{if}{}^0 + \Sigma_{f^c} \, p(\omega) \, \frac{L_{if}(I_f, \omega)}{L_f(I_f, \omega)} \tag{A.5}$$

Claim 2: $D_2 p_f \, x_{if}{}^0 = \alpha_{if} \, D_2 S_f{}^{I0}$

Proof. Recall that

$$p_f(I_f, k) = \frac{1}{N_f + n_f(I_f, k)} S_f{}^I(I_f, k)$$

It follows that

$$D_2 p_f = \frac{1}{N_f + n_f} D_2 S_f{}^I - \frac{D_2 n_f}{(N_f + n_f)^2} S_f{}^I$$

$$= \frac{1}{N_f + n_f} \left(D_2 S_f{}^I - \frac{D_2 n_f}{N_f + n_f} S_f{}^I \right)$$

Next, since

$$S_f{}^{I0} = \frac{N_f}{N_f + n_f} S_f{}^I$$

Direct Calculation shows that

$$D_2 S_f{}^{I0} = \frac{N_f}{N_f + n_f} D_2 S_f{}^I - \frac{N_f \, D_2 n_f}{(N_f + n_f)^2} S_f{}^I$$

$$= \frac{N_f}{N_f + n_f} \left(D_2 S_f{}^I - \frac{D_2 n_f}{(N_f + n_f)} S_f{}^I \right)$$

If follows that

$$D_2 p_f \, x_{if}{}^0 = \frac{x_{if}{}^0}{N_f + n_f} \left(D_2 S_f{}^I - \frac{D_2 n_f}{N_f + n_f} S_f{}^I \right) = \alpha_{if} \, D_1 S_f{}^{I0}$$

Q.E.D.

Since the claim holds the derivative in (A.5) is equivalent to

$$\alpha_{if} \, D_2 S_f{}^{I0} + \beta_{if} \, D_2 W_f{}^I$$

Therefore, the objective function may be expressed as

$$\alpha_{if} \, S_f{}^{IO} + \beta_{if} \, W_f{}^I = \alpha_{if}(S_f{}^{IP} - I_f) + (\beta_{if} - \alpha_{if}) \, W_f{}^I$$

Q.E.D.